

community is due to a lacuna of alternative means for obtaining bounded-error subsea XY position.

Although the advent of the global positioning system (GPS) provides bounded-error terrestrial navigation for both surface and air vehicles, seawater is opaque to the radio frequencies upon which GPS relies and, thus, GPS cannot be used by submerged underwater vehicles. Though ultra-short-baseline (USBL) acoustic navigation systems are preferred for short-range navigation, they are of limited usefulness for long-range navigation (Singh, Catipovic, Eastwood, Freitag, Henriksen, et al., 1996; Smith & Kronen, 1997) and, furthermore, also suffer from a TDMA update problem similar to that of LBL. Hence, neither GPS nor USBL solves the underwater vehicle XY navigation issue as outlined.

The high cost and power consumption of inertial navigation systems (INSs) has, until now, precluded their widespread use in nonmilitary undersea vehicles. Compact (<10-in. diameter), low-cost (<\$150,000), low-power (<15 W) INSs have become commercially available within the past decade (Gaiffe, 2002; Larsen, 2000a), offering an alternative method for instrumenting absolute XYZ displacement. Modern INS position error is on the order of 1% of path length and, when properly calibrated and aided by a Doppler velocity log (DVL), can approach better than 0.1% of path length. However, as with any open-loop navigation system, error accumulation is monotonic and hence INS alone is inadequate to support the needs of long-range, bounded-error navigation. For example, the path length of a vehicle traveling 3 kn for 48 h is 144 nm (266 km), resulting in an INS position error of 1.4 nm (2.6 km). This may be unacceptably large, depending on the mission requirements. For a survey of current underwater navigation technologies, the reader is referred to Kinsey, Eustice, and Whitcomb (2006).

In this paper we pursue the development of a synchronous-clock acoustic modem-based navigation system capable of supporting multivehicle bounded-error navigation over linear length scales not practically covered by traditional fixed-beacon LBL navigation means. Our navigation framework employs the WHOI Micro-Modem (Freitag, Grund, Partan, Ball, Singh, et al., 2005; Freitag, Grund, Singh, Partan, Koski, et al., 2005), an underwater acoustic modem developed by Woods Hole Oceanographic Institution (WHOI), in conjunction with commercial off-the-shelf (COTS), low-power stable clocks to yield a navigation system capable of intervehicle communication and one-way-travel-time (OWTT) ranging.

Previous work using modems and synchronous-clock navigation has been reported in Bahr and Leonard (2006) and Curcio, Leonard, Vaganay, Patrikalakis, Bahr, et al. (2005) for autonomous surface kayaks. In their work, each kayak is equipped with a tethered WHOI Micro-Modem and GPS receiver, which they use to provide a common time base for intervehicle synchronous ranging. Because

the kayaks are always on the water surface, their work did not have to address the challenges associated with implementing a synchronous-clock navigation framework on submerged vehicle nodes (where the use of GPS is precluded). Using an early predecessor of the modern WHOI Micro-Modem, synchronous-pinger OWTT navigation was reported in Singh, Bellingham, Hover, Lerner, Moran, et al. (2001), where integrated range-rate positioning was used for autonomous underwater vehicle (AUV) docking. In that work, the absolute initial estimate of AUV position was obtained using an LBL fix and was then subsequently updated using integrated range-rate measurements from the synchronous pinger to provide a dock-relative navigation correction.

Nonmodem synchronous-clock OWTT ranging has been reported in Hunt et al. (1974) for “in-hull navigation” of the manned deep-submergence vehicle Alvin. The navigation system consisted of two moored, precisely timed, pinging reference beacons synchronized to a master clock on Alvin. Reception of the transmitted beacon pulses, as measured with respect to Alvin’s master clock, allowed the submersible to measure the OWTT to each beacon and hence triangulate to get a position fix. Other nonmodem synchronous-clock navigation technology is that of the RAFOS oceanographic floats (Rossby, Dorson, & Fontaine, 1986), which employ precisely timed acoustic transmissions over very long acoustic paths to provide a position update once every 2 h (or so) with a navigation error approaching 1 s (1,500 m) over periods of several months. Other related work involving single-transponder, two-way-travel-time underwater navigation has been reported in Baccou and Jouvencel (2002), Gadre and Stilwell (2004, 2005), Larsen (2000b), Ross and Jouffroy (2005), Scherbatyuk (1995), and Vaganay, Baccou, and Jouvencel (2000).

In the terrestrial vehicle community there are examples of related work in range-only tracking and navigation that are tangentially applicable to our OWTT navigation paradigm. Song (1999) provides a Cramèr–Rao bound observability analysis of the range-only target tracking problem for radar and shows that if the target is moving at a constant acceleration, the observer must move with nonzero jerk to satisfy observability conditions. Similarly, Ristic, Arulampalam, and McCarthy (2002) extend the Cramèr–Rao analysis to include the availability of not only range but also range rate and provide a comparison between a maximum likelihood estimate (MLE), extended Kalman filter (EKF), and regularized particle filter—concluding that the noncausal, batch MLE estimator yields the best performance. Djughash and Singh (2008) devised a range-only simultaneous localization and mapping (SLAM) algorithm for radio-frequency identification (RFID) tags using a polar-representation EKF and demonstrated it to work with or without a priori knowledge of the RFID beacon locations. Also related is the work of Zhou and Roumeliotis

(2008), who devised an algebraic solution to three-degree-of-freedom (DOF) relative-pose estimation using vehicle-derived odometry and range-only observations between pairs of ground robots. They showed that minimally three range measurements are required to recover three-DOF robot relative pose and that a unique solution is obtained with five or more range constraints. This work is most similar to our own in the type of measurement constraints that it considers (range only and vehicle odometry); however, from their simulations Zhou and Roumeliotis showed that the algebraic solution produced large errors when corrupted with 10-cm-level range noise and hence would likely perform poorly with the type of range error considered in our underwater acoustic scenario. Also important when considering the applicability of terrestrial methods to the underwater scenario is the bandwidth-limited nature of the acoustic communication channel and its nonnegligible propagation delay, both of which are several orders of magnitude poorer than that of electromagnetic (EM)-based communication (Partan, Kurose, & Levine, 2007). EM waves are strongly attenuated underwater, and hence the assumption of reliable, instantaneous, fast communication between nodes in the network is not applicable.

In the following we describe our development of a synchronous-clock acoustic modem-based navigation system for multivehicle bounded-error navigation. Our contribution lies in the development of a novel time-synchronized navigation methodology, the principled formulation of how to model the OWTT constraints within a maximum likelihood estimation framework, and the experimental validation of the end-to-end OWTT methodology with an AUV in a real-world setting. The rest of the paper is organized as follows. Section 2 describes our synchronous-clock implementation and methodology. Sections 3 and 4 present a sensor fusion framework for stochastically analyzing OWTT pseudo-range measurements with strap-down onboard vehicle Doppler odometry for bounded-error navigation. Section 5 reports field results for two different sets of data collected in situ by a bottom-lock, Doppler-aided AUV accompanied with OWTT pseudo-range measurements acquired from a GPS-equipped surface ship. Finally, Section 6 closes with concluding remarks.

2. SYNCHRONOUS-CLOCK ACOUSTIC NAVIGATION

Most acoustic navigation systems are based on measuring two-way time-of-flight ranges whereby the navigation cycle consists of a vehicle-initiated interrogation pulse followed by a reply from all passively listening transponders (Hunt et al., 1974; Milne, 1983). In these systems, *each* vehicle in the acoustic network must interrogate the network in order to obtain a two-way time-of-flight measurement between it and all replying nodes. The advantage of such a system is that no absolute time base is required for time-of-flight measurement; the disadvantage is that when using

a narrowband acoustic navigation system, the overall update rate for each vehicle *decreases* as $1/N$ when deploying an N vehicle network. This is because narrowband designs must use a TDMA schedule to avoid acoustic interference (though commercial wideband systems capable of supporting multiple simultaneous channels have recently become available).

In comparison, accurate OWTT ranging can be determined by knowing precisely the transmit and receive times of an underwater acoustic communications packet. The result is a direct one-way time-of-flight measurement from source to receiver. The advantage over two-way-travel-time ranging is that OWTT ranging readily scales to a multivehicle environment within a master/slave architecture. This is because when a source (master) node interrogates the network, all receiving (slave) nodes can *passively* measure the one-way time of flight between each of them and the source node. Hence, in an N -vehicle environment, the overall update rate for each vehicle remains constant. The disadvantage, however, is increased complexity in hardware design because all nodes must carry their own synchronized stable clock hardware.

2.1. Methodology

Our methodology is to use OWTT ranging capabilities in the context of a surface ship acting as a moving beacon (Figure 1). For this purpose, we employ the synchronous-transmission capabilities of the WHOI Micro-Modem (Freitag, Grund, Partan, et al., 2005; Freitag, Grund, Singh, et al., 2005; Singh, Grund, Bingham, Eustice, Singh, et al., 2006). The synchronous-transmission feature of the WHOI Micro-Modem allows it to directly and accurately measure time of arrival (TOA) to within $125 \mu\text{s}$ (i.e., 18.75-cm range discretization at 1500-m/s sound speed) between a source and receiver using a user-supplied external pulse per second (PPS) reference clock. This common synchronous time base allows for a synchronous modem communication/navigation system whereby navigation data packets can encode time of origin information as well as local navigation data such as the sender's XY and depth positional data and associated error metric. Navigation packets can be broadcast to the vehicle network, allowing all receiving nodes to passively measure the elapsed time of flight between source node and destination. The OWTT pseudo-range knowledge, when used in conjunction with the decoded navigation data and other onboard vehicle navigation data, provides a mechanism for bounded-error self-localization.

In our method, a ship maneuvers with an AUV fleet, tending to vehicle launch/recovery support, while also acting as a global navigation aid by broadcasting GPS-derived ship-transducer position to the vehicle network. All vehicles that are within acoustic range of the ship and that passively receive the ship navigational data can then use this knowledge to compute a running position fix and

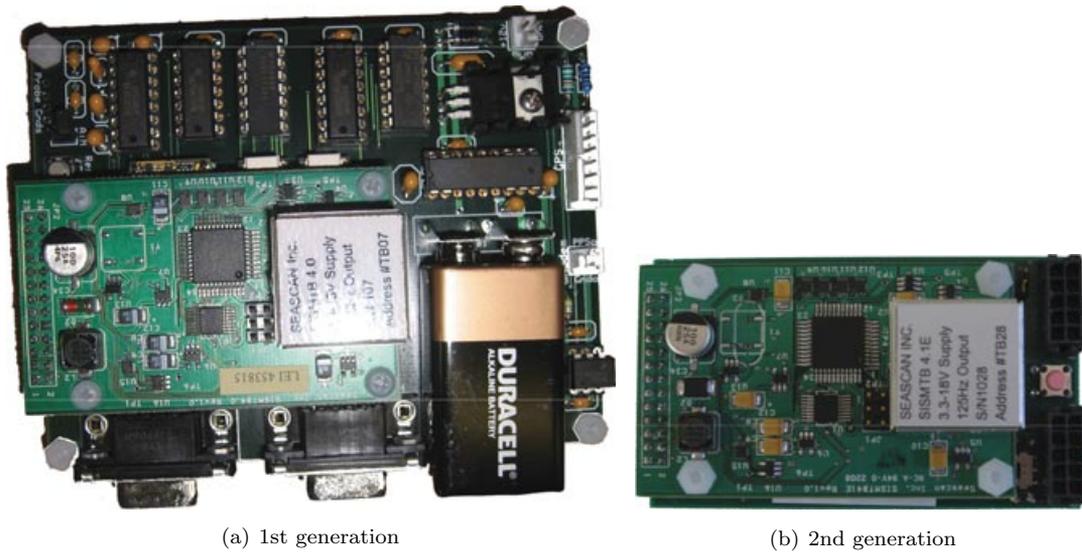


Figure 2. A depiction of a first- and second-generation PPSBOARD designed by the authors. The PPSBOARD integrates a COTS temperature-compensated crystal oscillator with a microcontroller-based printed circuit board. The PPSBOARD allows for higher level clock functionality such as PPS synchronization control, measuring of pre- and postdrive clock drift, and generating output suitable for real-time clock stabilization on the host PC via the open-source NTP algorithm.

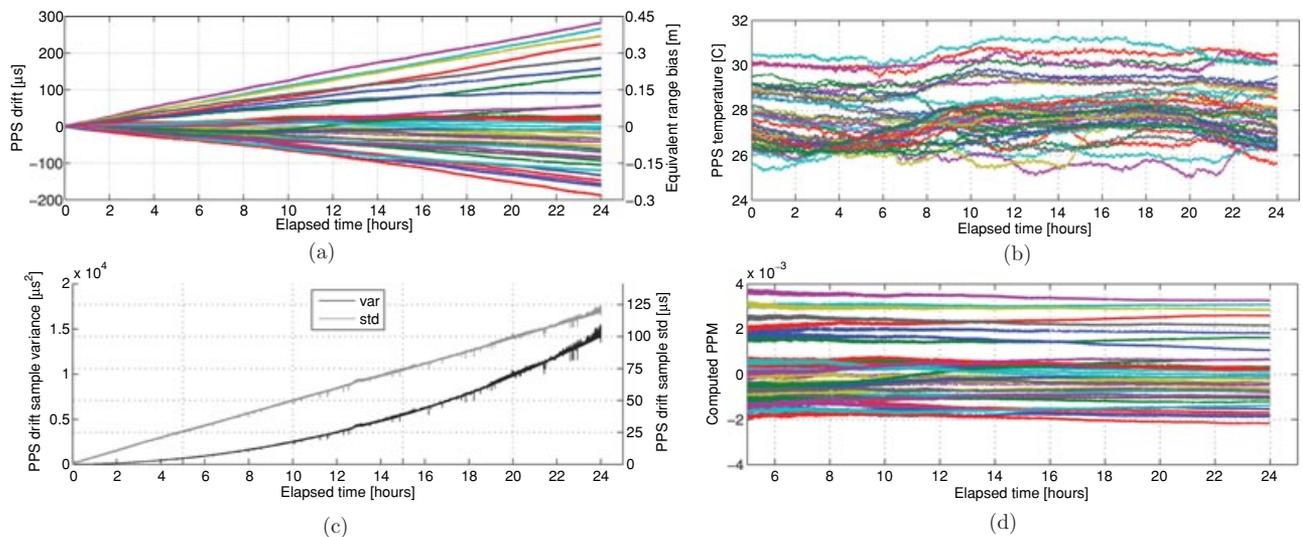


Figure 3. PPS drift characterization; benchmark results for 40 individual trials. Each trial was conducted over a 24-h period at ambient room temperature. The time axis shows the elapsed time in hours since synchronization of the PPSBOARD with respect to a reference drift-free PPS. (a) Measured PPS drift for all trials; each sample path is color coded by trial number. The y axis on the left reports the measured offset in microseconds, and the axis on the right reports it in range equivalent meters (assuming a sound speed of 1,500 m/s). (b) Measured onboard TXCO temperature; fluctuation is about normal ambient room temperature. (c) PPS drift sample variance and standard deviation across all trials. (d) Measured PPS drift for all trials, reported in units of parts per million (PPM).

displays measured drift, sample statistics, and recorded ambient temperature for each run. In particular, note in Figure 3(a) the time-varying drift phenomenon of the TXCO PPS. Qualitatively, the time-varying nature of the sample paths may, at first glance, appear similar to that of a random-walk process; however, note that each trial steadily increases in drift (positively or negatively) without significant random fluctuation. Hence, it does not exhibit Brownian motion. Furthermore, Figure 3(c) quantitatively displays the associated sample variance and standard deviation across the trials, where we see that the sample variance does not exhibit a linear growth with time as would be the case if it were indeed a random-walk process. Instead it appears to exhibit quadratic growth with time (linear standard deviation). At this point in time, further analysis is required to properly model clock drift.

The maximum drift error across all trials in Figure 3(a) is under 300 μs , which translates into a drift-induced range bias of less than 0.45 m over a 24-h period. Whereas the experiments reported here were for an ambient-temperature environment, we have also conducted some preliminary studies in which we immersed the TXCO into a controlled-temperature bath—exposing it to a step-response temperature change. The experimental setup consisted of synchronizing the PPSBOARD to the Meinberg at ambient room temperature (approximately 24°C) and then inserting the PPSBOARD into a temperature-controlled refrigerator set at approximately 4°C and allowing it to stabilize. We noted the PPSBOARD's drift relative to the Meinberg reference PPS and observed that the magnitude of error drift remained within the manufacturer's reported error tolerance of 20 ns/s. The manufacturer's technical specification for the TXCO is 20 ns/s (typical) over the calibrated temperature range of -5 to 35°C. This is relevant because PPS synchronization will typically occur at the surface prior to launch, meaning that the vehicle electronics and enclosed TXCO will go through a temperature gradient between ambient air at the surface to seafloor water temperature at depth. So whereas we should expect the drift performance of the TXCO to degrade with respect to the controlled laboratory results reported here, it still suggests that for relatively short-duration missions (i.e., several hours in length), drift-induced range bias will be a negligent error source in overall underwater vehicle navigation.

3. MAXIMUM LIKELIHOOD FUSION FRAMEWORK

In this section, we describe a maximum likelihood sensor fusion framework for bounded-error XY vehicle navigation that combines (1) vehicle-derived interping Doppler odometry, (2) ship GPS-derived position, and (3) OWTT pseudo-range measurements between ship and vehicle. The framework is suitable for offline batch postprocessing for the purposes of optimal renavigation and can also be extended to in situ vehicle use by selecting an appropriate sliding time window of most recent data.

3.1. Assumptions

In the forthcoming formulation we make the following assumptions:

1. First, that dives are relatively short in length (i.e., on the order of several hours) such that PPS drift-induced range bias remains negligible over the course of the dive and, therefore, can be neglected.
2. Second, that the sound speed profile is locally homogeneous within the prescribed bounding box of vehicle operations. This implies that time-of-flight measurements can be converted to pseudo-ranges via a linear scaling. In practice this is satisfied by conducting a pre-dive local conductivity–temperature–depth (CTD) cast and using a depth-weighted averaged sound velocity for scaling.
3. Third, that the estimation problem can be reduced to that of XY horizontal plane dynamics only. For this purpose, we assume that the vehicle is equipped with a pressure depth sensor of sufficient accuracy such that slant-range pseudo-ranges can be projected onto the horizontal plane. This also requires that the ship's trajectory never pass directly over top the vehicle, which would otherwise introduce a singularity into the horizontal range projection.
4. Fourth, that the vehicle is capable of measuring its own dead-reckoned (DR) XY odometry and error covariance in between OWTT pings. For example, this could be obtained from bottom-lock Doppler velocity data, an on-board INS system, or a vehicle dynamic model.
5. Finally, in the current formulation, we assume that all OWTT pseudo-range measurements occur between surface ship and vehicle only. At present, we do not consider intervehicle OWTT ranging. Hence, it is sufficient that each vehicle track ship trajectory only for the purposes of self-localization.

3.2. State Description

We denote topside ship trajectory as $\mathbf{x}_s(t) = [x_s(t), y_s(t)]^T$, where $[x_s(t), y_s(t)]$ represents XY ship transducer position in a locally defined, Earth-tangent, Cartesian coordinate frame. Similarly, we denote bottom-side vehicle trajectory as $\mathbf{x}_v(t) = [x_v(t), y_v(t)]^T$. In the aforementioned scenario, the ship initiates a OWTT broadcast at time t_s , and the vehicle receives that same broadcast at a corresponding later time $t_v > t_s$. For a given OWTT range measurement z_{r_i} , indexed by subscript i , we note that this measurement actually corresponds to a ship/vehicle sample pair, \mathbf{x}_{s_i} and \mathbf{x}_{v_i} , each sampled at distinct times $\mathbf{x}_{s_i} = \mathbf{x}_s(t_{s_i})$ and $\mathbf{x}_{v_i} = \mathbf{x}_v(t_{v_i})$, respectively. For notational convenience, we drop the explicit dependence on time and instead implicitly embed sample time within the \mathbf{x}_{s_i} and \mathbf{x}_{v_i} sample index notation. Figure 4 depicts a sequence of OWTT pseudo-range measurements occurring between ship and vehicle.

Next, we write our measurement observation models in terms of this sample index state description.

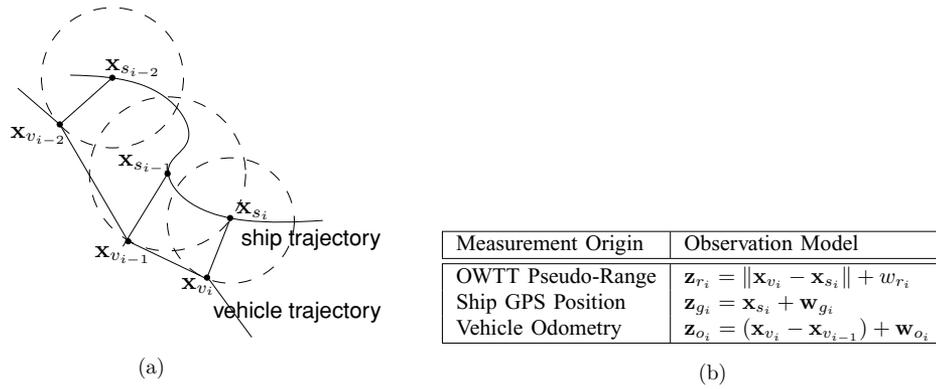


Figure 4. Maximum likelihood estimate formulation. (a) A depiction of the two-dimensional OWTT range geometry where \mathbf{x}_{s_i} and \mathbf{x}_{v_i} denote corresponding samples from the ship and vehicle trajectories, respectively. (b) A table of the available measurements and their relation to state entries. The w_{r_i} , \mathbf{w}_{g_i} , and \mathbf{w}_{o_i} terms represent additive measurement noise.

3.2.1. OWTT Pseudo-Range Observation Model

As previously stated, we use measured vehicle depth to horizontally project our raw time-of-flight pseudo-range measurements to the XY horizontal plane. Using this simplification, we write the horizontal range measurement as

$$z_{r_i} = \|\mathbf{x}_{v_i} - \mathbf{x}_{s_i}\| + w_{r_i}, \quad (1)$$

where w_{r_i} is an additive noise term that accounts for measurement error. For the current exposition we model w_{r_i} as being normally distributed with $w_{r_i} \sim \mathcal{N}(0, \Sigma_{r_i})$ with $E[w_{r_i} w_{r_j}] = 0$ for all $i \neq j$.

3.2.2. GPS-Derived Ship Position Observation Model

We first transform raw GPS measured latitude and longitude to the locally referenced XY coordinate system and then linearly interpolate to sample time t_{s_i} . On the basis of this, we write our ship GPS observation model as

$$\mathbf{z}_{g_i} = \mathbf{x}_{s_i} + \mathbf{w}_{g_i}, \quad (2)$$

where \mathbf{w}_{g_i} is an additive noise term that accounts for measurement error. We model this as being normally distributed with $\mathbf{w}_{g_i} \sim \mathcal{N}(0, \Sigma_{g_i})$ and $E[\mathbf{w}_{g_i} \mathbf{w}_{g_j}^T] = 0$ for all $i \neq j$.

3.2.3. Vehicle-Derived Odometry Observation Model

Vehicle-derived odometry measurements represent the interper vector displacement between OWTT fixes and are a necessary constraint in order to establish vehicle observability when dealing with single-transponder ranges (Gadre & Stilwell, 2004; Ross & Jouffroy, 2005). We write our odometry observation model as

$$\mathbf{z}_{o_i} = \mathbf{x}_{v_i} - \mathbf{x}_{v_{i-1}} + \mathbf{w}_{o_i}, \quad (3)$$

where \mathbf{w}_{o_i} is an additive noise term that accounts for measurement error. We model the error as being normally distributed with $\mathbf{w}_{o_i} \sim \mathcal{N}(0, \Sigma_{o_i})$ and $E[\mathbf{w}_{o_i} \mathbf{w}_{o_j}^T] = 0$ for all $i \neq j$.

3.3. Maximum Likelihood Estimator

We pose sensor fusion as a MLE optimization problem. In this context, we treat ship/vehicle sample pairs as unknown parameters that we wish to estimate. To begin, we consider the case of n range measurements and define the following:

$$\begin{aligned} \mathbf{X}_v &= \{\mathbf{x}_{v_i}\}_0^{n-1}, & \text{the set of vehicle trajectory samples,} \\ \mathbf{X}_s &= \{\mathbf{x}_{s_i}\}_0^{n-1}, & \text{the set of ship trajectory samples,} \\ \mathbf{Z}_r &= \{z_{r_i}\}_0^{n-1}, & \text{the set of pseudo-range measurements,} \\ \mathbf{Z}_g &= \{\mathbf{z}_{g_i}\}_0^{n-1}, & \text{the set of ship GPS measurements,} \\ \mathbf{Z}_o &= \{\mathbf{z}_{o_i}\}_1^{n-1}, & \text{the set of odometry measurements,} \end{aligned}$$

with $\mathbf{X} = \{\mathbf{X}_v, \mathbf{X}_s\}$ and $\mathbf{Z} = \{\mathbf{Z}_r, \mathbf{Z}_g, \mathbf{Z}_o\}$.

Denoting the measurement likelihood as $L(\mathbf{X})$, we have

$$L(\mathbf{X}) = p(\mathbf{Z}|\mathbf{X}) \quad (4a)$$

$$= p(\mathbf{Z}_r|\mathbf{X}_v, \mathbf{X}_s) p(\mathbf{Z}_g|\mathbf{X}_s) p(\mathbf{Z}_o|\mathbf{X}_v), \quad (4b)$$

where the mutual independence of measurements on parameters is explicit. To optimize, we wish to find

$$\hat{\mathbf{X}} = \arg \max_{\mathbf{X}} p(\mathbf{Z}|\mathbf{X}), \quad (5)$$

which is equivalent to solving

$$\hat{\mathbf{X}} = \arg \min_{\mathbf{X}} -\ln p(\mathbf{Z}|\mathbf{X}). \quad (6)$$

Under the assumed observation models and noise statistics, we can write our objective function, $C(\mathbf{X})$, as

$$\begin{aligned}
 C(\mathbf{X}) &= -\ln p(\mathbf{Z}|\mathbf{X}) \\
 &= \frac{1}{2} \sum_{i=0}^{n-1} (z_{r_i} - \|\mathbf{x}_{v_i} - \mathbf{x}_{s_i}\|)^{\top} \Sigma_{r_i}^{-1} (z_{r_i} - \|\mathbf{x}_{v_i} - \mathbf{x}_{s_i}\|) \\
 &\quad + \frac{1}{2} \sum_{i=0}^{n-1} (\mathbf{z}_{g_i} - \mathbf{x}_{s_i})^{\top} \Sigma_{g_i}^{-1} (\mathbf{z}_{g_i} - \mathbf{x}_{s_i}) \\
 &\quad + \frac{1}{2} \sum_{i=1}^{n-1} [\mathbf{z}_{o_i} - (\mathbf{x}_{v_i} - \mathbf{x}_{v_{i-1}})]^{\top} \Sigma_{o_i}^{-1} [\mathbf{z}_{o_i} - (\mathbf{x}_{v_i} - \mathbf{x}_{v_{i-1}})].
 \end{aligned} \tag{7}$$

Furthermore, this can be written compactly as

$$C(\mathbf{X}) = [\mathbf{Z} - \mathbf{h}_Z(\mathbf{X})]^{\top} \Sigma_Z^{-1} [\mathbf{Z} - \mathbf{h}_Z(\mathbf{X})], \tag{8}$$

where $\mathbf{h}_Z(\mathbf{X})$ is the stacked vector of observations and Σ_Z is the block-diagonal measurement covariance.

The above formulation amounts to a nonlinear weighted least-squares optimization problem. To solve, we employ the Levenberg–Marquardt algorithm (Press, Teukolsky, Vetterling, & Flannery, 1992) starting with an initial guess of DR vehicle position and GPS-measured ship position. A first-order estimate of parameter covariance Σ_X , can then be obtained by evaluating the Jacobian at the optima:

$$\begin{aligned}
 \mathbf{J} &= \left. \frac{d\mathbf{h}_Z(\mathbf{X})}{d\mathbf{X}} \right|_{\hat{\mathbf{x}}} \\
 \text{and computing (Haralick, 1998)} \\
 \Sigma_X &= (\mathbf{J}^{\top} \Sigma_Z^{-1} \mathbf{J})^{-1}.
 \end{aligned} \tag{9}$$

4. DVL-DERIVED ODOMETRY COVARIANCE

The MLE formulation assumes that measurement covariances are available. For GPS measurements, most receivers are capable of reporting a quality of fix, which can be used to derive a covariance estimate; for OWTT ranges, we can estimate TOA precision by measuring the sample jitter between two fixed nodes. The only remaining item is to assign an appropriate covariance measure to our interping odometry samples. In the following, we derive a theoretical first-order calculation of covariance for bottom-lock DVL XY displacement.

4.1. Theoretical First-Order Covariance Estimate

Integrated Doppler XY displacement between sample times $t_{v_{i-1}}$ and t_{v_i} , under the assumption of negligible pitch and roll, can be written as

$$\mathbf{x}_{o_{i-1,i}} = \int_{t_{v_{i-1}}}^{t_{v_i}} \mathbf{R}(\psi(t)) \mathbf{u}(t) dt, \tag{10}$$

where $\psi(t)$ is the time-varying vehicle heading, $\mathbf{u}(t) = [u(t), v(t)]^{\top}$ is the vector velocity body-frame surge and sway, and

$$\mathbf{R}(\theta) = \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix}$$

is the orthonormal body to world rotation matrix. In practice, the DVL returns discrete-time samples; therefore, we rewrite Eq. (10) as a summation using forward Euler integration as

$$\mathbf{x}_{o_{i-1,i}} = T \sum_{j=0}^{m-1} \mathbf{R}(\psi_j) \mathbf{u}_j, \tag{11}$$

where T is the DVL sample period, $m = \lfloor (t_{v_i} - t_{v_{i-1}}) / T \rfloor$ is the number of interping samples, $\mathbf{u}_j = \mathbf{u}(t_{v_{i-1}} + jT)$ is the sampled velocity, and $\psi_j = \psi(t_{v_{i-1}} + jT)$ is the sampled heading.

To proceed with a calculation of DVL displacement covariance, we model the heading and velocity as each being corrupted by zero-mean, mutually independent, white Gaussian noise processes $w_{\psi_j} \sim \mathcal{N}(0, \sigma_{\psi_j}^2)$ and $\mathbf{w}_{\mathbf{u}_j} \sim \mathcal{N}(0, \Sigma_{\mathbf{u}})$, respectively, which leads to a DVL odometry measurement of

$$\mathbf{z}_{o_i} = T \sum_{j=0}^{m-1} \mathbf{R}(\psi_j + w_{\psi_j}) (\mathbf{u}_j + \mathbf{w}_{\mathbf{u}_j}). \tag{12}$$

Assuming that the distribution on w_{ψ_j} is sufficiently narrow, we make the small angle approximation:

$$\begin{aligned}
 \cos(\psi_j + w_{\psi_j}) &\approx \cos \psi_j - w_{\psi_j} \sin \psi_j, \\
 \sin(\psi_j + w_{\psi_j}) &\approx \sin \psi_j + w_{\psi_j} \cos \psi_j,
 \end{aligned}$$

and apply it within the DVL odometry model as

$$\mathbf{z}_{o_i} \approx T \sum_{j=0}^{m-1} [\mathbf{R}(\psi_j) - w_{\psi_j} \mathbf{R}'(\psi_j)] (\mathbf{u}_j + \mathbf{w}_{\mathbf{u}_j}) \tag{13a}$$

$$\begin{aligned}
 &= T \underbrace{\sum_{j=0}^{m-1} \mathbf{R}(\psi_j) \mathbf{u}_j}_{\mathbf{x}_{o_{i-1,i}}} + T \underbrace{\sum_{j=0}^{m-1} \mathbf{R}(\psi_j) \mathbf{w}_{\mathbf{u}_j}}_{S_1} \\
 &\quad - T \underbrace{\sum_{j=0}^{m-1} w_{\psi_j} \mathbf{R}'(\psi_j) \mathbf{u}_j}_{S_2} - T \underbrace{\sum_{j=0}^{m-1} w_{\psi_j} \mathbf{R}'(\psi_j) \mathbf{w}_{\mathbf{u}_j}}_{S_3}
 \end{aligned} \tag{13b}$$

$$= \mathbf{x}_{o_{i-1,i}} + S_1 - S_2 - S_3, \tag{13c}$$

where

$$\mathbf{R}'(\theta) = \begin{bmatrix} -\sin \theta & -\cos \theta \\ \cos \theta & -\sin \theta \end{bmatrix}.$$

Taking the expectation of Eq. (13b), we have $E[\mathbf{z}_{o_i}] = \mathbf{x}_{o_{i-1,i}}$. To compute the DVL covariance we evaluate

$$\Sigma_{o_i} = E[(\mathbf{z}_{o_i} - E[\mathbf{z}_{o_i}])(\mathbf{z}_{o_i} - E[\mathbf{z}_{o_i}])^\top] \quad (14a)$$

$$\begin{aligned} &= E[S_1 S_1^\top] - E[S_1 S_2^\top] - E[S_1 S_3^\top] - E[S_2 S_1^\top] \\ &\quad + E[S_2 S_2^\top] + E[S_2 S_3^\top] - E[S_3 S_1^\top] \\ &\quad + E[S_3 S_2^\top] + E[S_3 S_3^\top] \end{aligned} \quad (14b)$$

and note that the zero terms arise from the mutual independence of w_{ψ_j} and \mathbf{w}_{u_j} . Evaluating the expectations in Eq. (14) and defining U_j as

$$U_j = \begin{bmatrix} u_j^2 & u_j v_j \\ u_j v_j & v_j^2 \end{bmatrix},$$

we arrive at the final expression for first-order DVL odometry covariance:

$$\begin{aligned} \Sigma_{o_i} &= T^2 \sum_{j=0}^{m-1} R(\psi_j) \Sigma_{u_j} R(\psi_j)^\top + T^2 \sum_{j=0}^{m-1} R'(\psi_j) U_j R'(\psi_j)^\top \sigma_{\psi_j}^2 \\ &\quad + T^2 \sum_{j=0}^{m-1} R'(\psi_j) \Sigma_{u_j} R'(\psi_j)^\top \sigma_{\psi_j}^2. \end{aligned} \quad (15)$$

4.2. Linear Trackline Example

To help the reader gain some intuition behind the interpretation of Eq. (15), here we consider the case of constant forward velocity under constant heading and no side slip:

$$\mathbf{u}_j = [u, 0]^\top, \quad \psi_j = \psi.$$

This configuration replicates the typical survey scenario of linear tracklines. For the purposes of demonstration, we further assume time-invariant noise statistics, and in the case of Doppler velocity, isotropic covariance:

$$\sigma_{\psi_j}^2 = \sigma_{\psi}^2, \quad \Sigma_{u_j} = \sigma_d^2 I_{2 \times 2}.$$

Under the above scenario, Eq. (15) reduces to

$$\begin{aligned} \Sigma_{o_i} &= m T^2 \sigma_d^2 (1 + \sigma_{\psi}^2) I_{2 \times 2} \\ &\quad + m T^2 u^2 \sigma_{\psi}^2 \begin{bmatrix} \sin^2 \psi & \sin \psi \cos \psi \\ \sin \psi \cos \psi & \cos^2 \psi \end{bmatrix}. \end{aligned} \quad (16)$$

The first term in Eq. (16) accounts for an isotropic linear growth with time in XY DVL trajectory error. This error source is due to the random walk contribution of measured velocity error, typical of an independent increments process. The second term in Eq. (16) accounts for heading uncertainty and is dependent on direction of travel—causing error growth transverse to the direction of motion. For example, if heading due north (i.e., $\psi = 0$), then this term accounts for accumulated error growth in the east–west direction due to measurement uncertainty in the true heading of

the vehicle. Figure 5(a) demonstrates this phenomenon for a representative set of headings where the ellipsoids have been calculated for 10 m of travel at 1-m/s forward velocity with 4-Hz sampling at 3-mm/s velocity precision and a heading precision of 1 deg. The ellipsoids represent 1- σ contours and have been inflated by a factor of 1,000 for visualization.

To gain further insight into the magnitude of heading error and its effect on XY uncertainty, Figure 5(b) displays results of evaluating Eq. (16) for a sampling of typical heading and velocity errors. For this demonstration, we set $\psi = 90$ deg for a path length of 100 m at representative speeds of $u = 1, 3, \text{ and } 5$ m/s and velocity standard deviations of $\sigma_d = 3, 4, \text{ and } 5$ mm/s, respectively. These numbers are provided by the RDI Workhorse manual as being typical 1,200-kHz bottom-lock precisions (RD Instruments, 2001). Most noticeably, Figure 5(b) demonstrates that cross-track error is the dominating component for heading precisions above 1 deg.

5. RESULTS

In this section we report results for a set of two field experiments employing OWTT navigation with MLE sensor fusion. Both experiments were for a two-node configuration consisting of the tending surface ship and a single AUV. In each case, the SeaBED AUV platform (Figure 6) was used.

5.1. Experimental Setup

SeaBED is instrumented with a typical suite of oceanographic navigation sensors including pressure sensor depth, 1,200-kHz DVL body-frame velocities, an IXSEA north-seeking three-axis fiber-optic gyrocompass for attitude, and a PPS-capable WHOI Micro-Modem (Singh, Can, Eustice, Lerner, McPhee, et al., 2004). In addition, we integrated our PPSBOARD and a Garmin GPS-16HVS GPS receiver into the vehicle system so that we could conduct OWTT experiments. The Garmin GPS unit outputs a 1-PPS reference signal accurate to within 1 μ s of UTC when it has GPS lock. We use this PPS reference for predictive time synchronization of the free-running TXCO onboard the AUV while at the surface.

The surface ship is equipped with a PPS-capable Micro-Modem, a GPS receiver used for measuring ship transducer position, and a Meinberg GPS-based NTP time server as a stable clock reference. The Meinberg unit outputs a 1-PPS signal accurate to within 100 ns and is drift free.

5.2. Experimental Results

5.2.1. Experiment 1: GPS Validation

During December 2005, we operated the SeaBED AUV offshore the coast of Woods Hole, Massachusetts, using the research vessel (R/V) Tioga. For this set of experiments we

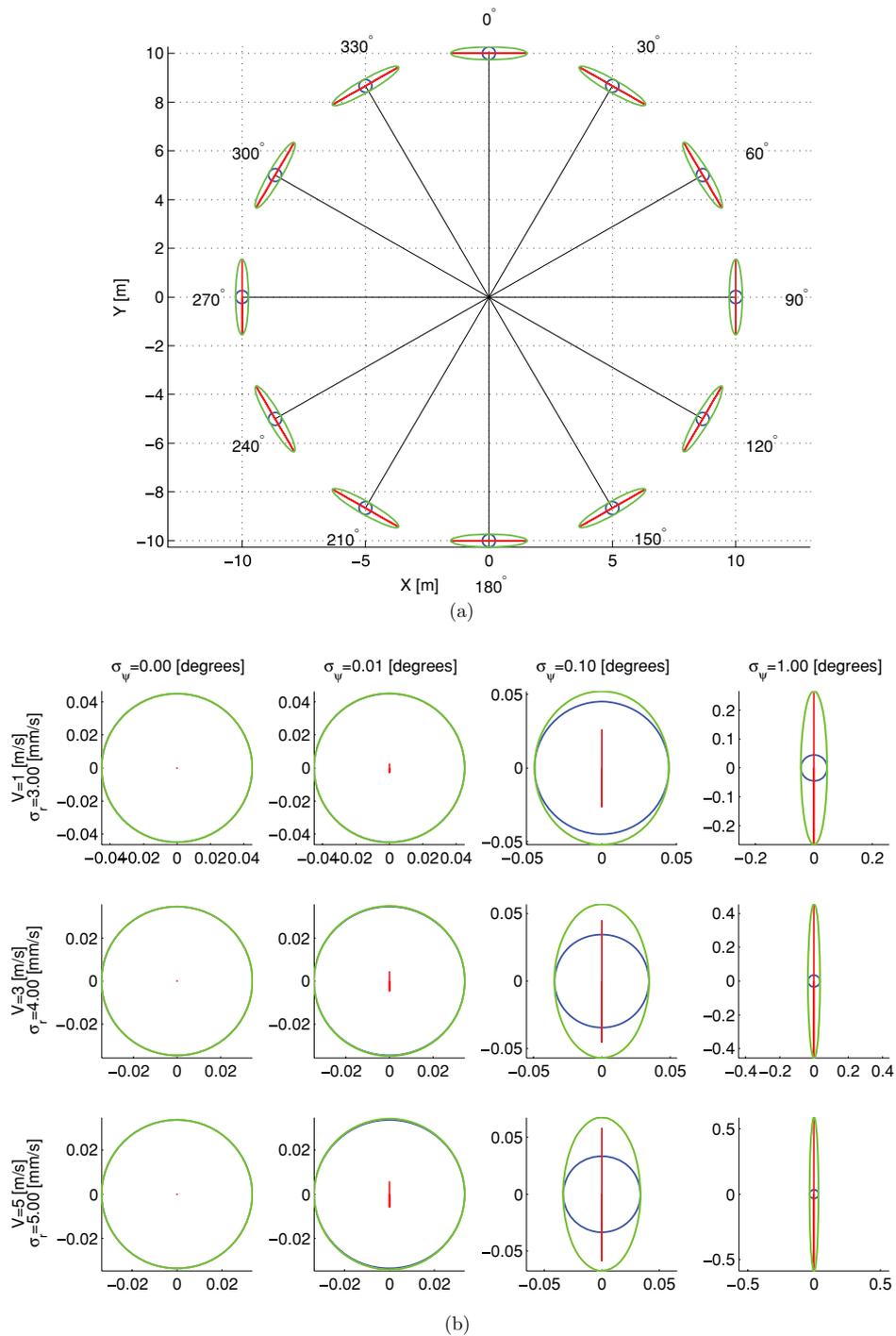


Figure 5. A demonstration of theoretical XY DVL odometry covariance for constant forward velocity with no side slip under constant heading. (a) The DVL covariance ellipsoid for a representative set of headings. The ellipsoids represent 1- σ contours and have been inflated by a factor of 1,000 for visualization. In blue we have the isotropic component of covariance error, in red the anisotropic component due to heading error, and in green their superposition. Note that the heading error contributes to error growth transverse to the direction of travel. (b) The theoretical DVL covariance ellipsoid for a range of typical heading precisions and 1,200-kHz DVL velocity precisions for 100 m of path length in the east–west direction. Units shown are in meters, and the ellipsoids represent 3- σ contours. Refer to (a) for ellipse colors.



Figure 6. The SeaBED AUV as used during field experiments (Singh et al., 2004). The orange Tracklink beacon mounted on the front strut provided an independent measurement of ship-to-vehicle range.

deployed the AUV in approximately 15 m of water and programmed the vehicle to execute a trajectory comprising two 100-m concentric boxes at a forward speed of 0.4 m/s. In this environment, the AUV had Doppler bottom lock from the surface with a total mission time of approximately 1.5 h; PPS clock drift onboard the vehicle was less than 4 μ s (Figure 7). The ship remained on anchor and OWTT ranges were broadcast at a periodic schedule of every 5, then 20, s. Figure 8 displays the raw OWTT data recorded between the AUV and ship; for comparison, independently measured two-way-travel-time (Tracklink USBL) ranges are shown. Equation (15) was used to calculate interping DVL odometry covariance using settings of 3-mm/s standard deviation for the body-frame surge/sway velocities (assumed isotropic) and 0.1-deg standard deviation for the

heading; these numbers were obtained from the respective manufacturer’s manuals as being typical precisions. We set the OWTT ranges at 18.75-cm standard deviation, based on the Micro-Modem’s 125- μ s TOA detection resolution. For ship GPS position error, we used the horizontal error estimated by the receiver and assumed it to be isotropic.

Figure 9 displays results from the MLE fusion of the OWTT pseudo-ranges and bottom-lock DVL odometry. The raw DVL trajectory, shown in blue, was obtained by forward Euler integration, using the GPS drop position of the AUV as the origin. Shown in red is the globally referenced MLE-derived trajectory; this result was postprocessed offline. Shown in green is the independently measured postdive GPS position of the AUV, which cross validates the OWTT result. Figure 9(b) provides a plot of GPS referenced error versus time, showing good agreement between OWTT-derived position and onboard GPS.

5.2.2. Experiment 2: LBL Validation

In July 2006, we operated aboard the Greek vessel R/V Aegeo as part of a joint WHOI/Hellenic Centre for Marine Research (HCMR) research expedition in the Mediterranean. This time we performed a longer series of OWTT experiments with the SeaBED AUV, deploying it for a mission just under 2 h in duration. The water depth was approximately 50 m, and the programmed survey trajectory consisted of two grid patterns: one oriented east-to-west and the other north-to-south. The survey bounding box was approximately 200 m on a side. We used the same measurement noise parameters as described in Section 5.2.1. In addition, we deployed a two-transponder LBL net for independent position validation.

Figure 10 shows results comparing raw DVL, MLE-fused OWTT data, and cross-validated 12-kHz LBL vehicle position. For this experiment the surface ship free drifted about the survey site, occasionally motoring to get back on station; its trajectory is depicted by the cyan curve in Figure 10(a). Figure 10(b) shows a close-up view of

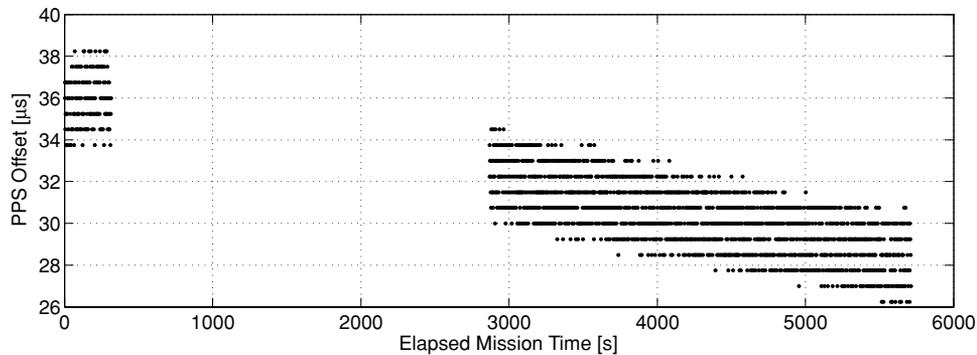


Figure 7. Pre- and postdive measured TXCO PPS offset with respect to GPS-derived absolute PPS time base. The gap in the time series corresponds to when the AUV was subsea. During the course of the dive, the TXCO drifted by approximately 4 μ s from an initial offset of 36 μ s to 32 μ s.

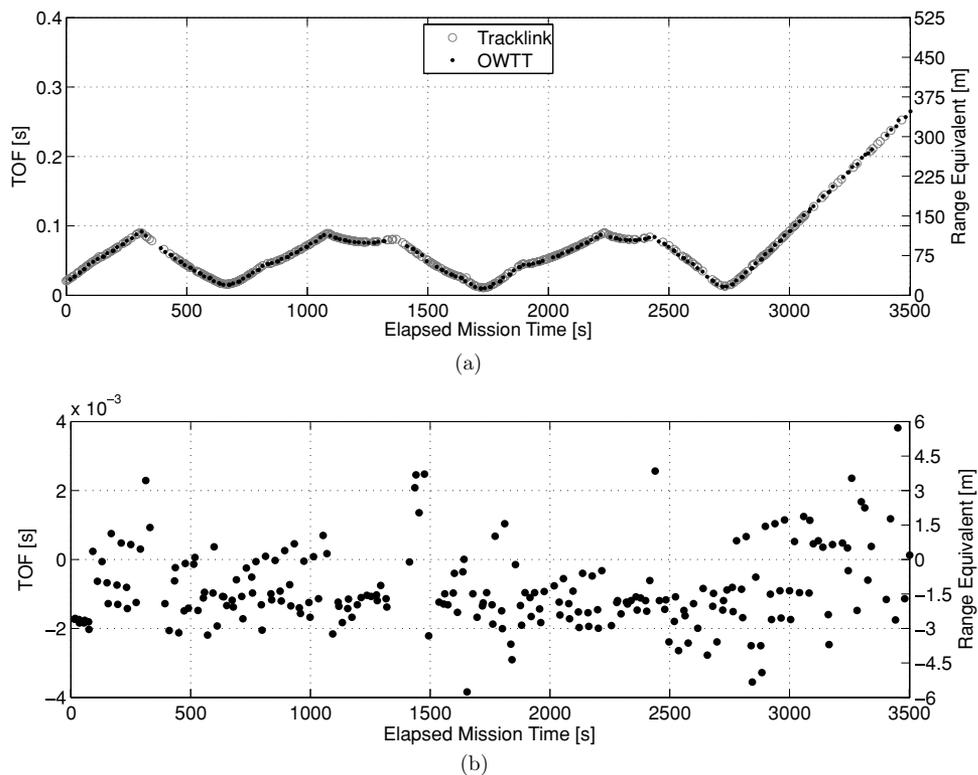


Figure 8. A comparison of modem-derived OWTT ranges versus USBL two-way-travel-time ranges measured between ship and AUV. (a) A plot of raw ranges as measured by the the Tracklink USBL system and modem-derived OWTT. (b) A comparison of the time-of-flight discrepancy obtained by linearly interpolating the USBL range data to the OWTT time base. For reference, the y-axis scale on the right shows the equivalent range error assuming a sound speed of 1,500 m/s.

the renavigated survey. The coarse agreement between OWTT MLE and LBL is visually evident, in contrast with the raw Doppler track, whose error increases with time. Figures 10(c) and 10(d) and 10(e) and 10(f) show that the raw DVL trackline exhibits a time-dependent bias, whereas the OWTT renavigated trajectory does not.

5.3. Discussion

The experimental results from the two field tests corroborate that the DVL-aided OWTT navigation framework outperforms the DVL-alone navigation error—overall showing good agreement with the bounded-error, independent positioning methods provided by surface GPS and LBL. In particular, the time-cumulative DVL drift bias has been removed via the fusion of the OWTT range constraints in the MLE framework. What is interesting to note is that no single OWTT range measurement sufficiently constrains the vehicle's position; rather each OWTT measurement places the vehicle on an annulus centered about the ship's broadcast position. It is through the vector displacement odometry provided by the DVL that three or more OWTT range measurements provide sufficient constraint to uniquely XY

bound the position error of the vehicle. In the absence of bottom-lock DVL, the vector displacement odometry constraint could be derived from an INS or, with less fidelity, from a vehicle dynamic model; however, we have not yet independently corroborated the performance of either of those two alternative scenarios.

During both experimental tests, the communication protocol was symmetric in that the vehicle periodically transmitted OWTT packets up to the ship, while the ship frequently downlinked OWTT packets to the vehicle (both on a predefined schedule). In the proposed MLE estimation framework, this makes no difference in the optimal post-processed renavigation of the AUV because OWTT measurements are indexed between *pairs* of ship/vehicle states and hence can accommodate such an uplink/downlink scenario. For real-time subsea navigation of the AUV, a sliding time-window version of the batch MLE could be run on the vehicle using downlink OWTT measurements and navigational data to compute a running position fix. We are also currently pursuing a more computationally efficient recursive estimation formulation based on an EKF framework, and its dual—an extended information filter, as preliminarily reported in Webster, Eustice, Singh, and

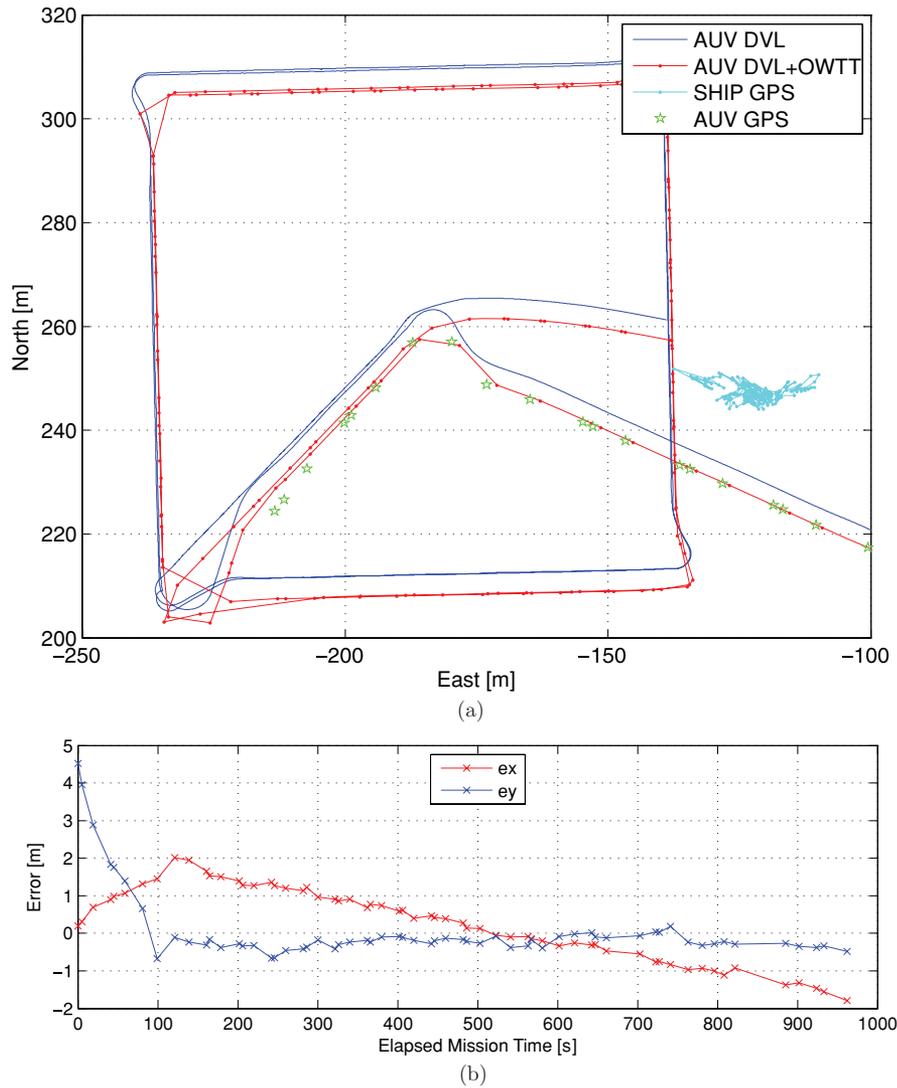


Figure 9. MLE sensor fusion results for a two-node network consisting of a bottom-lock, Doppler-aided AUV and a GPS-equipped surface ship. Each node is equipped with a WHOI Micro-Modem and onboard stable PPS source for synchronous-clock communication/navigation. (a) The AUV swam two colocated box trajectories (100 m per side), each at a different depth set point while the ship remained anchored. Shown in blue is the raw DVL-derived AUV trajectory; in cyan is the GPS-derived ship position; in red is the globally referenced MLE AUV trajectory; and in green is the end-of-dive GPS-measured AUV position, which serves as an independent ground truth. (b) The error between end-of-dive OWTT-derived and GPS-measured AUV position.

Whitcomb (2009) and Webster, Whitcomb, and Eustice (2010), respectively. Note that for efficient simultaneous multivehicle navigation, the ship should act as a master and primarily downlink to the passively listening vehicle fleet because under this scenario all passively receiving vehicles within acoustic range will measure the OWTT constraint independent of the number of vehicles in the water.

Another interesting point is that the relative ship/vehicle trajectory geometry plays an important role in the global positional observability obtained by

OWTT constraints from a single moving reference beacon. Earlier work by Song (1999) for the range-only target tracking problem showed that the observer's trajectory must exhibit nonzero derivative dynamics of the target's trajectory (i.e., if the target is traveling on a constant velocity trajectory, the observer's trajectory must exhibit nonzero acceleration). In practice, this means that the ship's trajectory should be nonzero and should change its heading course if a vehicle is conducting a linear trackline survey. Further study into optimal trajectory geometries for minimal global error is the subject of future work.

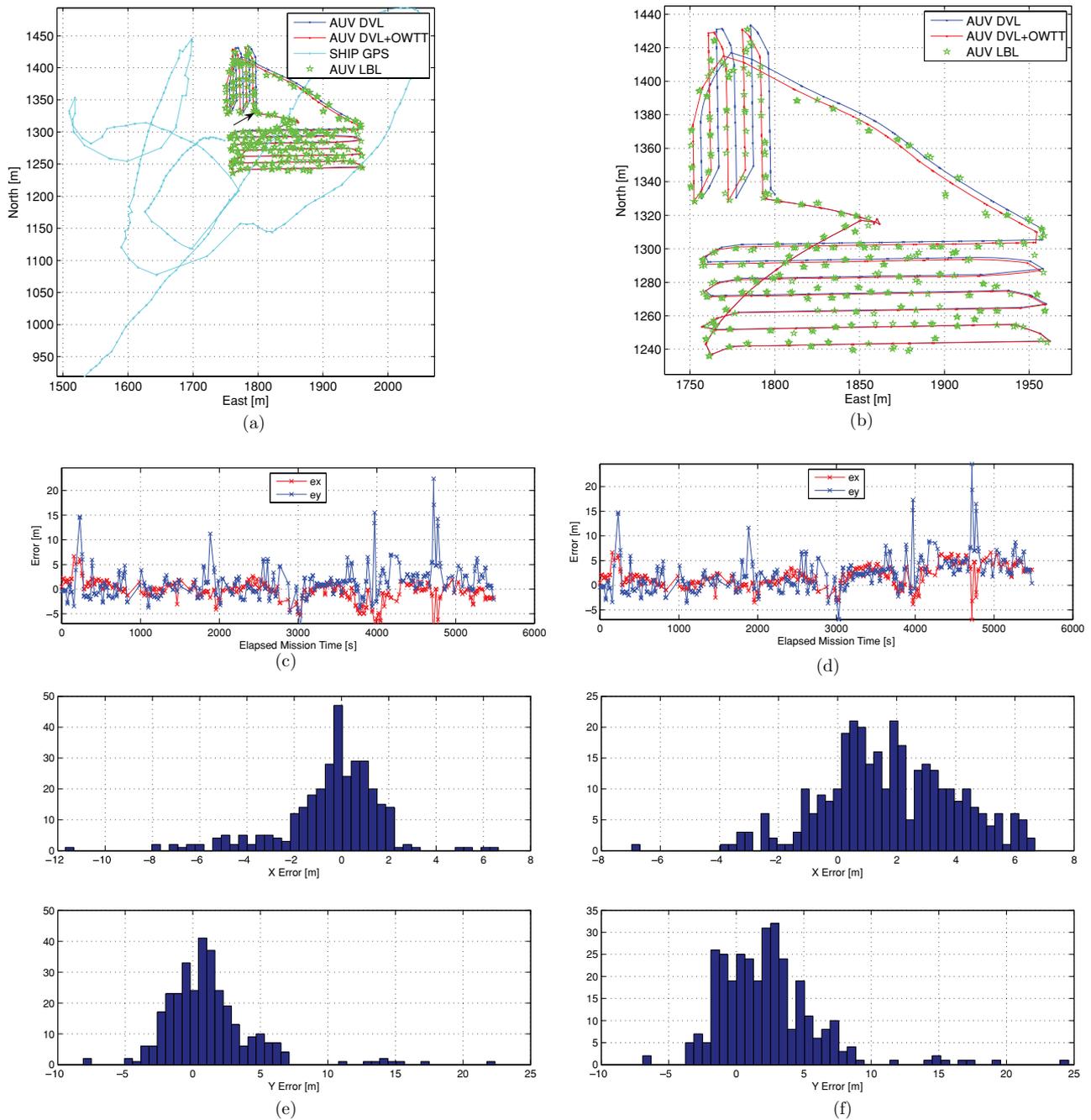


Figure 10. MLE sensor fusion results for a two-node configuration consisting of a GPS-equipped surface ship and bottom-lock, Doppler-aided AUV. The survey took place in approximately 50 m of water. (a) The programmed vehicle trajectory consisted of two grid-survey patterns: one oriented east-to-west and the other north-to-south. Shown in blue is the raw DVL-derived DR AUV trajectory; in cyan is the GPS-derived ship position; in red is the globally referenced MLE AUV trajectory; and in green is the 12-kHz LBL-derived AUV position, which serves as an independent ground truth. The mission starts and ends at approximately (1794,1330), as indicated by the arrow. (b) A close-up view of the vehicle trajectory. The discrepancy between raw DVL position and LBL increases with time. In contrast, the OWTT-aided trajectory agrees well with the independently measured LBL position. (c) and (d) OWTT and raw DVL position error versus elapsed mission time; error is defined with respect to LBL. Again, note the zero-mean nature of the OWTT-derived position versus the time-dependent DVL error. X and Y error histograms are shown in (e) and (f).

Also worth noting is that for dives exceeding 1 day in length, which are becoming typical of modern oceanographic AUVs, the modeling of clock drift will play a crucial role in the effective use of the OWTT navigation framework. The experiments reported herein were of short duration, and hence clock drift could be ignored. On dives exceeding 14 h in length, we can expect clock drift on the TXCO's employed in our system to approach 1 ms of time (worst case) and thereby introduce a range bias of 1.5 m into the pseudo-range constraint. This bias would be exhibited in all range measurements and strongly time correlated—therefore it may be possible to observe clock bias within the state estimation framework. Notionally, one simple solution would be to stop the ship and vehicle and exchange OWTT modem messages down and up, and by comparing the downlink OWTT to the uplink OWTT, one could estimate the clock offset. In the case of either one or both vehicles moving, this would become more difficult to measure as it would require knowledge of the range change due to intervehicle motion to be compensated for using odometry or other kinematic information. At this point in time, further study is required to investigate the conditions under which clock-bias estimation may be feasible.

6. CONCLUSIONS

This article reported recent advances in the development of a synchronous-clock acoustic communication/navigation system for underwater vehicles. The long-term goal of this work is to enable the task of navigating a fleet of AUVs over geometrically unconstrained length scales with bounded-error commensurate with standard 12-kHz LBL navigation. Toward this end, we have reported the development of a low-power, stable clock system suitable for integration on AUVs. We have also formulated a principled maximum likelihood fusion framework for combining OWTT pseudo-range measurements with vehicle odometry for bounded-error navigation. For this purpose we presented a covariance analysis of bottom-lock, Doppler-derived odometry. Results from two field experiments validating the OWTT maximum likelihood estimator framework were reported. Future research in this area will address the modeling of PPS clock drift, the use of water-lock DVL velocity odometry, and a decentralized estimation framework to support multivehicle navigation with intervehicle ranging.

ACKNOWLEDGMENTS

This work was supported by the National Science Foundation (NSF) under Award Numbers ATM-0427220, ATM-0428122, and IIS-0746455 and in part by the National Oceanic and Atmospheric Administration (NOAA) Office of Ocean Exploration under Grant Number NA06OAR4600092.

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